OPEN ARCHITECTURE GRADIENT COIL SET FOR MAGNETIC RESONANCE IMAGING APPARATUS

CROSS-REFERENCE TO RELATED APPLICATION

This application claims the benefit of U.S. provisional patent application Serial No. 60/270,960 filed February 22, 2001.

BACKGROUND OF THE INVENTION

The present invention relates to magnetic resonance imaging and, in particular, to gradient coils used therewith.

A Magnetic Resonance Scanner where the direction of the main static field is orthogonal to the pole surface and to the magnet's main symmetry plane is commonly called an Open Magnet System.

Previous methods for production of linear magnetic gradients in such MRI systems consist of winding discrete coils, in a bunched or distributed fashion on an electrically insulating hollow right cylinder coil-form, and driving said coil with a limited voltage. Conventional bunched coil designs are the Maxwell and Modified Maxwell Pair for the Z-gradient coil (parallel to the main magnetic filed), and the Golay or Modified Golay (multi-arc) Saddle coils for the X/Y gradient coils. Previous methods consisted of iteratively placing elemental loops or arcs on the cylindrical coil form until the desired gradient strength, gradient uniformity and inductance were achieved. In general, these types of designs constituted as method referred as a "forward approach" whereby a set of initial coil positions (i.e. initial current distribution) were defined. The gradient field, the energy/inductance were calculated and if they did not fall within specified design criteria, the positions of the coils will be shifted (statistically or otherwise) and the new outcome will be re-evaluated. This iterative procedure will continue until a suitable solution that satisfies all the design criteria is found.

An alternative approach in the design of gradient coils is referred as an "inverse approach". According to this methodology, a set of predetermined constraint points for the gradient magnetic field is chosen inside the desired imaging volume. Via a Lagrange minimization technique involving the stored magnetic energy or power of the structure, the

1 appropriate continuous current distribution that generates the predetermined gradient field inside

2 the imaging volume is derived. Using a stream function discretization technique, the continuous

3 current distribution of the coil can be approximated by a set of discrete current loops that are

4 connected in series and share the same constant current. In order to ensure that the discretization

5 process is valid, the gradient field is regenerated using the Biot-Savart law to the discrete current

6 distribution and compare the results with the ones of the analytic approach. If the error is

7 relatively small, the discrete current distribution is accepted as a valid one.

8 Open magnet geometries with vertically directed fields have been a successful presence

9 in the MRI arena due to the inherent open architecture scheme that promotes patient comfort and

10 diminishes patient claustrophobia. Recently, vertical field systems with higher main field

11 strengths (0.5 T to 1.0 T) were also introduced in an attempt to improve image quality, as well as

12 to expand the spectrum of clinical applications, especially fast imaging techniques not available

13 on lower field strength MR scanners. The necessity for utilizing such imaging techniques is

14 strongly dependent upon the availability of a suitable gradient system capable of delivering high

15 gradient field strengths with a significant improvement to the gradient coil's slew rate. For open

16 magnet geometries with vertically directed fields, biplanar gradient coils have been the geometry

17 of choice, since there are conformed to the main magnet's geometric shape and assist in

18 improving patient comfort and reducing patient claustrophobia. There is however a drawback in

19 regards to the biplanar designs. Because of the demand that the coil must have enough gap to

20 allow for clear access to the human torso, the distance between the two planes is excessively

21 high. Increasing the gap between the two planes reduces the efficiency of the biplanar gradient in

22 a level. Thus for a whole body gradient coil set, it is very difficult to generate gradient fields of

23 the order of 50-60 mT/m with slew rates approaching 1000 T/m/sec. An alternative solution to

24 this issue is the introduction of uniplanar gradient coil designs, which offer superb gradient field

25 characteristics over a very limited imaging volume. Uniplanar gradients are mainly utilized on

26 local imaging applications and are not recommended for applications with sizable FOV such as

27 the one covering the human head. One viable alternative for performing ultra fast imaging of the

28 human head for a vertical field system is the use of insertable biplanar gradients in a very close

29 proximity to the human head. Due to the reduction of the gap between the planes the efficiency

30 of the coil is significantly improved, and such a design is capable of generating high peak and

31 high uniformity gradient fields over a volume similar to the size of the human head (25 cm

1   DSV). But such a design is prohibitive because it violates the two principles of which the open

2   magnet designs are based on. Placing the two planes over the human head and especially over

3   the patient's eyes, it compromises the patient's comfort and enhances patient anxiety.

4   Gradient coil sets in common use make serious tradeoffs between coil performance and

5   maintaining the "open," non-claustrophobic configuration, desired in open MRI systems.

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7   SUMMARY OF THE INVENTION

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9   A MRI gradient coil set includes a uniplanar Z-gradient coil, a biplanar X-gradient coil,

10 and a biplanar Y-gradient coil. The coil set provides an open Z-axis face.

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12   BRIEF DESCRIPTION OF THE DRAWINGS

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14   FIG. 1 is a perspective view of a possible gradient coil set according to the invention.

15   FIG. 2a is a table showing on-axis linearity.

16   FIG. 2b is a table showing off-axis uniformity.

17   FIG. 3 is a table showing on-axis field behavior.

18   FIG. 4 is a table showing on-axis field behavior.

19   FIG. 5 is a gradient pattern.

20   FIG. 6 is a gradient pattern.

21   FIG. 7a is a gradient pattern.

22   FIG. 7b is a gradient pattern.

23   FIG. 8 is a table showing on-axis field behavior.

24   FIG. 9 is a gradient pattern.

25   FIG. 10 is a gradient pattern.

26   FIG. 11 is a gradient pattern.

27   FIG. 12 is a gradient pattern.

28   FIG. 13 is a gradient pattern.

29   FIG. 14 is a table showing on-axis behavior.

30   FIG. 15 is a gradient pattern.

31   FIG. 16a is a gradient pattern.

1   FIG. 16b is a gradient pattern.

2   FIG. 17a is a gradient pattern.

3.   FIG. 17b is a gradient pattern.

4   FIG. 18 is a perspective view of a possible gradient coil set according to the invention

5 with shielded biplanar coils.

6   FIG. 19 is a perspective view of a possible gradient coil set according to the invention

7 with unshielded biplanar coils.

8   FIG. 20 is a perspective view of a possible gradient coil set according to the invention

9 with shielded biplanar coils and incorporating radio frequency coils.

10   FIG. 21 is a perspective view of a possible gradient coil set according to the invention

11 with unshielded biplanar coils and incorporating radio frequency coils.

12   FIG. 22 is a perspective view of a possible gradient coil set according to the invention in

13 an array configuration.

14   FIG. 23 is a perspective view of a possible gradient coil set according to the invention

15 incorporating knee and upper thigh frequency coils.

16   FIG. 24 is a perspective view of a possible gradient coil set according to the invention

17 incorporating knee and foot radio frequency coils.

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19   DESCRIPTION OF THE PREFFERRED EMBODIMENTS

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21   In this invention, a new alternative design for an insertable open architecture gradient coil

22 set is given which is focused on enhancing patient comfort and abolishing patient anxiety, while

23 delivering a gradient field with high peak strength and improved slew rate. According to this

24 architecture, the Z gradient coil is a uniplanar design that is placed in parallel with the main

25 magnet's poles. In order to form a complete gradient set, the two transverse (X, Y) gradients are

26 generated by a pair of biplanar designs which are placed perpendicular to the magnets poles, but

27 they may be physically mounted on the same side with the uniplanar Z gradient. The gradient

28 field generated from the two transverse coils exhibits similar peak and slew rate performances as

29 the uniplanar Z gradient coil over a comparable imaging volume. When this coil is designated for

30 head imaging applications, parabolic cut-offs on the two side transverse biplanar coils may be

31 implemented in a symmetric fashion. Such an insertable coil design does not disrupt the vision of

1   the human subject and thus makes it more patient friendly without imposing restriction on the

2   field strength and quality. The proposed design can also be implemented in a self-shielded

3   configuration for reduction of induced eddy current effects on the vicinity of the main magnet's

4   poles. In addition, the proposed design can be paired with an additional set of gradient coils

5   (shielded or not) in an array configuration with a capability of extending the imaging coverage of

6   the gradient coil system, while improving patient comfort and diminishing patient

7   claustrophobia.

8

9   **Theory**

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11   FIG. 1, shows the geometrical shape for a self-shielded open architecture gradient set.

12   The gap between the two primary planes on the biplanar section of the coil is denoted as 2a while

13   the distance between the shielding planes of the biplanar section of the coil is denoted as 2b. For

14   the uniplanar section of the coil, the distance between the plane section of the coil and the

15   geometric center of the coil is denoted as a', while the distance between the geometric center of

16   the coil and shielding part of the biplanar section as b'. The schematic representation of a head

17   Transmit/Receive Radiofrequency coil is also included.

18

19   **Axial (Z) Uniplanar Gradient**

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21   Let us assume that the position of the plane for the primary gradient coil is at z=-a' from

22   the geometric center of the coil, while the position of the shielding coil is z=-b' from the center of

23   the coil. The current density distribution for this design is confined on the xy plane and has the

24   expression:

$$\vec{J}(x,y) = [J_x(x,y)\hat{x} + J_Y(x,y)\hat{y}] \qquad (1)$$

25

26

1 In addition the expression for the vector potential is given by:

2

$$A_x = \frac{\mu_0}{8\pi^2} \int \int_{-\infty}^{+\infty} d\alpha d\beta \frac{1}{\sqrt{\alpha^2+\beta^2}} e^{i\alpha x + i\beta y} \left[e^{-\sqrt{\alpha^2+\beta^2}(Z+a')} J_x^{-a'}(\alpha,\beta) + e^{-\sqrt{\alpha^2+\beta^2}(Z+b')} J_x^{-b'}(\alpha,\beta) \right] for\ z \geq -a' \quad (2)$$

$$A_y = \frac{\mu_0}{8\pi^2} \int \int_{-\infty}^{+\infty} d\alpha d\beta \frac{1}{\sqrt{\alpha^2+\beta^2}} e^{i\alpha x + i\beta y} \left[e^{-\sqrt{\alpha^2+\beta^2}(Z+a')} J_y^{-a'}(\alpha,\beta) + e^{-\sqrt{\alpha^2+\beta^2}(Z+b')} J_y^{-b'}(\alpha,\beta) \right] for\ z \geq -a' \quad (3)$$

3

4 And for the region outside the planes z≤-b', the expression for the vector potential is:

$$A_x = \frac{\mu_0}{8\pi^2} \int \int_{-\infty}^{+\infty} d\alpha d\beta \frac{1}{\sqrt{\alpha^2+\beta^2}} e^{i\alpha x + i\beta y} \left[e^{\sqrt{\alpha^2+\beta^2}(Z+a')} J_x^{-a'}(\alpha,\beta) + e^{\sqrt{\alpha^2+\beta^2}(Z+b')} J_x^{-b'}(\alpha,\beta) \right] for\ z \leq -b' \quad (4)$$

$$A_y = \frac{\mu_0}{8\pi^2} \int \int_{-\infty}^{+\infty} d\alpha d\beta \frac{1}{\sqrt{\alpha^2+\beta^2}} e^{i\alpha x + i\beta y} \left[e^{\sqrt{\alpha^2+\beta^2}(Z+a')} J_y^{-a'}(\alpha,\beta) + e^{\sqrt{\alpha^2+\beta^2}(Z+b')} J_y^{-b'}(\alpha,\beta) \right] for\ z \leq -b' \quad (5)$$

5 where $J_{(x,y)}(\alpha,\beta)$ represent the double Fourier Transform of $J_{(x,y)}(x,y)$, respectively. Since there

6 is no charge free in the vicinity of the gradient coils, the application of the current continuity

7 equation ($\nabla \bullet J = 0$) leads to a relationship between the components of the current density as:

8 $$J_y(\alpha,\beta) = -\frac{\alpha}{\beta} J_x(\alpha,\beta) \quad (6)$$

9 From equations (2)-(5) and equation (6), the expression for the vector potential at the location of

10 the two planes is given by:

$$A_x = \frac{\mu_0}{8\pi^2} \int \int_{-\infty}^{+\infty} d\alpha d\beta \frac{1}{\sqrt{\alpha^2+\beta^2}} e^{i\alpha x + i\beta y} \left[J_x^{-a'}(\alpha,\beta) + e^{-\sqrt{\alpha^2+\beta^2}(b'-a')} J_x^{-b'}(\alpha,\beta) \right] for\ z = -a' \quad (7)$$

$$A_y = \frac{\mu_0}{8\pi^2} \int \int_{-\infty}^{+\infty} d\alpha d\beta \frac{1}{\sqrt{\alpha^2+\beta^2}} e^{i\alpha x + i\beta y} \left(-\frac{\alpha}{\beta}\right) \left[J_x^{-a'}(\alpha,\beta) + e^{-\sqrt{\alpha^2+\beta^2}(b'-a')} J_x^{-b'}(\alpha,\beta) \right] for\ z = -a' \quad (8)$$

$$A_x = \frac{\mu_0}{8\pi^2} \int \int_{-\infty}^{+\infty} d\alpha d\beta \frac{1}{\sqrt{\alpha^2+\beta^2}} e^{i\alpha x + i\beta y} \left[e^{\sqrt{\alpha^2+\beta^2}(b'-a')} J_x^{-a'}(\alpha,\beta) + J_x^{-b'}(\alpha,\beta) \right] for\ z = -b' \quad (9)$$

$$A_y = \frac{\mu_0}{8\pi^2} \int \int_{-\infty}^{+\infty} d\alpha d\beta \frac{1}{\sqrt{\alpha^2+\beta^2}} e^{i\alpha x + i\beta y} \left(-\frac{\alpha}{\beta}\right) \left[e^{\sqrt{\alpha^2+\beta^2}(b'-a')} J_x^{-a'}(\alpha,\beta) + J_x^{-b'}(\alpha,\beta) \right] for\ z = -b' \quad (10)$$

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12 The expression for the magnetic field is:

$$B_z = -\partial_z A_y = \frac{i\mu_0}{8\pi^2} \int \int_{-\infty}^{+\infty} d\alpha d\beta \frac{\sqrt{\alpha^2+\beta^2}}{\beta} e^{i\alpha x + i\beta y} e^{-\sqrt{\alpha^2+\beta^2}(z+a')} J_x^{-a'}(\alpha,\beta)$$

$$\left[1 - e^{-2\sqrt{\alpha^2+\beta^2}(b'-a')}\right] \text{ for } z \geq -a' \qquad (11)$$

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2

3   and since the magnetic field is symmetric along the x and y directions and monotonically

4   increased along the z direction, equation (11) becomes:

$$B_z = -\frac{\mu_0}{8\pi^2} \int \int_{-\infty}^{+\infty} d\alpha d\beta \frac{\sqrt{\alpha^2+\beta^2}}{\beta} \cos(\alpha x) \cos(\beta y) e^{-\sqrt{\alpha^2+\beta^2}(z+a')} \hat{J}_x^{-a'}(\alpha,\beta) \left[1 - e^{-2\sqrt{\alpha^2+\beta^2}(b'-a')}\right] \qquad (12)$$

with

$$J_x^{-b'}(\alpha,\beta) = -e^{-\sqrt{\alpha^2+\beta^2}(b'-a')} J_x^{-a'}(\alpha,\beta) (\text{shielding condition } B_z = 0 \text{ for } z \geq -b') \qquad (13)$$

and

$$J_x^{-a'}(\alpha,\beta) = i\hat{J}_x^{-a'}(\alpha,\beta) \Rightarrow \hat{J}_x^{-a'}(\alpha,\beta) = 4\int \int_0^\infty dx\, dy \, \cos(\alpha x) \sin(\beta y) J_x^{-a'}(x,y) \qquad (14)$$

6   Furthermore, the expression of the stored magnetic energy is:

$$W_M = \frac{1}{2} \int_V d^3\vec{x} \quad \vec{A} \bullet \vec{J} = \frac{\mu_0}{16\pi^2} \int \int_{-\infty}^{+\infty} d\alpha\, d\beta \frac{\sqrt{\alpha^2+\beta^2}}{\beta^2} |J_x^{-a'}(\alpha,\beta)|^2 \left[1 - e^{-2\sqrt{\alpha^2+\beta^2}(b'-a')}\right] \qquad (15)$$

7   In order to find the optimum continuous current density, which will generate a gradient magnetic

8   field in predetermined locations inside the imaging volume, can be extracted via a Lagrange

9   optimization technique, According to this technique, a functional expression, which is a

10   quadratic function of the current density is constructed. An example of such an expression

11   includes which includes the stored magnetic energy and the magnetic field, and can have the

12   form:

$$F(J_x^{-a'}) = W_m - \sum_{j=1}^{N} \lambda_j \left[B_z(\vec{r}_j) - B_{zsc}(\vec{r}_j)\right] \qquad (16)$$

13   where λ_j are the Lagrange multipliers and $B_{zsc}(r_j)$ are the constrained values of the magnetic field

14   inside the desired imaging volume. Minimizing F with respect to $J_x^{-a'}$, a linear matrix equation

15   for $J_x^{-a'}$ is obtained:

$$J_x^{-a'} = -\beta \sum_{j=1}^{N} \lambda_j \cos(\alpha x_j)\cos(\beta y_j)e^{-\sqrt{\alpha^2+\beta^2}(z_j+a')} \qquad (17)$$

1   and the Lagrange multipliers are determined via the constrained equation :

$$\sum_{j=1}^{N} C_{ij}\lambda_j = B_{zSC_i} \Rightarrow C_{ij} = \frac{\mu_0}{2\pi^2} \int\limits^{+\infty} \int d\alpha d\beta \cos(ax_i)\cos(\beta y_i)e^{-\sqrt{\alpha^2+\beta^2}(z_i+a')}$$

$$\sum_{j=1}^{N} \lambda_j \cos(\alpha x_j)\cos(\beta y_j)e^{-\sqrt{\alpha^2+\beta^2}(z_j+a')}\left[1 - e^{-2\sqrt{\alpha^2+\beta^2}(b'-a')}\right] \qquad (18)$$

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3   **For the case of a non-shielded uniplanar axial gradient set, the above expressions are the**

4   **same at the limit when b′→∞.**

5

6   From the continuous current distribution and using employing the stream function technique, a

7   discrete set of loops that closely approximates the continuous current density pattern is

8   generated. In order to verify the accuracy of the discretization, the gradient field of the coil is re-

9   evaluated by applying the Biot-Savart law to the discrete current distribution.

10

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12   **Transverse X or Y Uniplanar Gradient**

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14   The expression for the magnetic field is from equation (11):

$$B_z = -\partial_z A_y = \frac{i\mu_0}{8\pi^2} \int \int_{-\infty}^{+\infty} d\alpha d\beta \frac{\sqrt{\alpha^2+\beta^2}}{\beta}e^{i\alpha x+i\beta y}e^{-\sqrt{\alpha^2+\beta^2}(z+a')}J_x^{-a'}(\alpha,\beta)$$

$$\left[1 - e^{-2\sqrt{\alpha^2+\beta^2}(b'-a')}\right] \text{ for } z \geq -a'$$

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16

17   and since the magnetic field is symmetric along the x and z directions and antisymmetric along

18   the y direction, equation (11) becomes:

$$B_z = -\frac{\mu_0}{8\pi^2} \int \int_{-\infty}^{+\infty} d\alpha d\beta \frac{\sqrt{\alpha^2 + \beta^2}}{\beta} \sin(\alpha x) \cos(\beta y) e^{-\sqrt{\alpha^2+\beta^2}(z+a')} \hat{J}_x^{-a'}(\alpha,\beta)\left[1 - e^{-2\sqrt{\alpha^2+\beta^2}(b'-a')}\right] \quad (12a)$$

with

$$J_x^{-b'}(\alpha,\beta) = -e^{-\sqrt{\alpha^2+\beta^2}(b'-a')} J_x^{-a'}(\alpha,\beta)(shielding \quad condition: B_z = 0 \, for \, z \geq -b') \quad (13a)$$

and

$$J_x^{-a'}(\alpha,\beta) = 4 \int \int_0^{\infty} dx\, dy\, \sin(\alpha x) \sin(\beta y) J_x^{-a'}(x,y) \quad (14a)$$

Furthermore, the expression of the stored magnetic energy is:

$$W_M = \frac{1}{2} \int_V d^3\vec{x} \quad \vec{A} \bullet \vec{J} = \frac{\mu_0}{16\pi^2} \int \int_{-\infty}^{+\infty} d\alpha\, d\beta \frac{\sqrt{\alpha^2+\beta^2}}{\beta^2} |J_x^{-a'}(\alpha,\beta)|^2 \left[1 - e^{-2\sqrt{\alpha^2+\beta^2}(b'-a')}\right] \quad (15a)$$

In order to find the optimum continuous current density, which will generate a gradient magnetic field in predetermined locations inside the imaging volume, can be extracted via a Lagrange optimization technique, According to this technique, a functional expression, which is a quadratic function of the current density is constructed. An example of such an expression includes which includes the stored magnetic energy and the magnetic field, and can have the form:

$$F(J_x^{-a'}) = W_m - \sum_{j=1}^{N} \lambda_j \left[B_z(\vec{r}_j) - B_{zsc}(\vec{r}_j)\right] \quad (16)$$

where λ_j are the Lagrange multipliers and $B_{zsc}(r_j)$ are the constrained values of the magnetic field inside the desired imaging volume. Minimizing F with respect to $J_x^{-a'}$, a linear matrix equation for $J_x^{-a'}$ is obtained:

$$J_x^{-a'} = -\beta \sum_{j=1}^{N} \lambda_j \sin(\alpha x_j) \cos(\beta y_j) e^{-\sqrt{\alpha^2+\beta^2}(z_j+a')} \quad (17a)$$

and the Lagrange multipliers are determined via the constrained equation :

$$\sum_{j=1}^{N} C_{ij}\lambda_j = B_{zSC_i} \Rightarrow C_{ij} = \frac{\mu_0}{2\pi^2} \int \int^{+\infty} d\alpha d\beta \sin(\alpha x_i) \cos(\beta y_i) e^{-\sqrt{\alpha^2+\beta^2}(z_i+a')}$$

$$\sum_{j=1}^{N} \lambda_j \sin(\alpha x_j) \cos(\beta y_j) e^{-\sqrt{\alpha^2+\beta^2}(z_j+a')} \left[1 - e^{-2\sqrt{\alpha^2+\beta^2}(b'-a')}\right] \quad (18a)$$

1 **For the case of a non-shielded uniplanar axial gradient set, the above expressions are the**

2 **same at the limit when b′→∞.**

3

4 From the continuous current distribution and using employing the stream function technique, a

5 discrete set of loops that closely approximates the continuous current density pattern is

6 generated. In order to verify the accuracy of the discretization, the gradient field of the coil is re-

7 evaluated by applying the Biot-Savart law to the discrete current distribution.

8

9 **Phased Array considerations: Mutual energy between two coils.**

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11 Let us assume a second planar set with planes locations of $z=-\bar{a}'$ and $z=-\bar{b}'$. If the second

12 set is shifted along the x or y directions or both by an amount of x_0 and y_0, respectively, the

13 analytical expression of the current density distribution is given by:

14

15

$$J_x^{-\bar{a}',-\bar{b}'}(\alpha,\beta) = e^{-i\beta y_0} e^{-i\alpha x_0} \int \int_{-\infty}^{\infty} dx \, dy \, e^{-i\beta(y-y_0)} e^{-i\alpha(x-x_0)} \, J_x^{-\bar{a}',-\bar{b}'}(x,y) \qquad (19)$$

16

17 Considering for simplicity of the evaluations, that the second set is shifted only along the y

18 direction by an amount of y_0. In this situation the mutual energy between these two modules is

19 given by:

20

21

$$W_{Mutual} = \frac{1}{2} \int_V d^3\vec{x} \quad \vec{A}^{(-a',-b')} \bullet \vec{J}^{(-\bar{a}',-\bar{b}')} = \frac{\mu_0}{4\pi^2} \int \int_{-\infty}^{+\infty} d\alpha \, d\beta \, \frac{\sqrt{\alpha^2+\beta^2}}{\beta^2} e^{-i\beta y_0} J_x^{-a'}(\alpha,\beta) J_x^{-\bar{a}'*}(\alpha,\beta)$$

$$e^{-\sqrt{\alpha^2+\beta^2}(a'-\bar{a}')} \left[1 - e^{-2\sqrt{\alpha^2+\beta^2}(b'-(a'-\bar{a}'))} \right] \qquad (20)$$

22

23 If both planes represent the same gradient coil axis, equation (20) becomes

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$$W_{Mutual} = \frac{\mu_0}{4\pi^2} \int \int_{-\infty}^{+\infty} d\alpha \, d\beta \, \frac{\sqrt{\alpha^2+\beta^2}}{\beta^2} \cos(\beta y_0) J_x^{-a'}(\alpha,\beta) J_x^{-\bar{a}'*}(\alpha,\beta)$$

$$e^{-\sqrt{\alpha^2+\beta^2}(a'-\bar{a}')} \left[1 - e^{-2\sqrt{\alpha^2+\beta^2}(b'-(a'-\bar{a}'))} \right] \qquad (21)$$

1 The optimum phased array configuration occurs at the y_0 points where the value of the mutual

2 inductance between the two sets is minimized.

3

4 **Transverse (X, Y) Biplanar Gradient Coils**

5

6 Let us assume that for the self-shielded biplanar design, the gap between the two planes of the

7 primary gradient coil is $y=2a$, while the gap between the planes of the shielding coil is $z=2b$.

8 The current density distribution for either the primary gradient coil or the secondary gradient coil

$$\vec{J}(x,y) = [J_x^{+a}(x,z)\hat{x} + J_z^{+a}(x,z)\hat{z}]\,\delta(y-a) + [J_x^{-a}(x,z)\hat{x} + J_z^{-a}(x,z)\hat{z}]\,\delta(y+a) +$$
$$[J_x^{+b}(x,z)\hat{x} + J_z^{+b}(x,z)\hat{z}]\,\delta(y-b) + [J_x^{-b}(x,z)\hat{x} + J_z^{-b}(x,z)\hat{z}]\,\delta(y+b) \quad (22)$$

9 is confined on the xz plane. The analytical expression for the current density of the coil is:

10 In addition the expression for the vector potential is given by:

11

12 where $J_{(x,z)}(\alpha,\beta)$ represent the double Fourier Transform of $J_{(x,z)}(x,z)$, respectively. Since there

$$A_{x,z} = \frac{\mu_0}{8\pi^2}\int\int_{-\infty}^{+\infty}d\alpha d\beta \frac{1}{\sqrt{\alpha^2+\beta^2}}e^{i\alpha x+i\beta z}\left[\begin{array}{c} e^{-\sqrt{\alpha^2+\beta^2}(a-y)}J_{x,z}{}^a(\alpha,\beta)+e^{-\sqrt{\alpha^2+\beta^2}(a+y)}J_{x,z}{}^{-a}(\alpha,\beta)+ \\ e^{-\sqrt{\alpha^2+\beta^2}(b-y)}J_{x,z}{}^b(\alpha,\beta)+e^{-\sqrt{\alpha^2+\beta^2}(b+y)}J_{x,z}{}^{-b}(\alpha,\beta) \end{array}\right] for\,|y|\le a \quad (23)$$

$$A_{x,z} = \frac{\mu_0}{8\pi^2}\int\int_{-\infty}^{+\infty}d\alpha d\beta \frac{1}{\sqrt{\alpha^2+\beta^2}}e^{i\alpha x+i\beta z}\left[\begin{array}{c} e^{-\sqrt{\alpha^2+\beta^2}(a-y)}J_{x,z}{}^a(\alpha,\beta)+e^{\sqrt{\alpha^2+\beta^2}(a+y)}J_{x,z}{}^{-a}(\alpha,\beta)+ \\ e^{-\sqrt{\alpha^2+\beta^2}(y-b)}J_{x,z}{}^b(\alpha,\beta)+e^{\sqrt{\alpha^2+\beta^2}(b+y)}J_{x,z}{}^{-b}(\alpha,\beta) \end{array}\right] for\,|y|\ge b \quad (24)$$

13 is no charge free in the vicinity of the gradient coils, the application of the current continuity

14 equation ($\nabla\bullet J=0$) leads to a relationship between the components of the current density as:

15 $J_z(\alpha,\beta) = -\frac{\alpha}{\beta}J_x(\alpha,\beta)$ \qquad (25)

16 From equations (23)-(24) and equation (25), the expression for the vector potential at the

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location of the two planes is given by:

$$A_x = \frac{\mu_0}{8\pi^2} \int \int_{-\infty}^{+\infty} \frac{d\alpha d\beta}{\sqrt{\alpha^2+\beta^2}} e^{i\alpha x + i\beta z} \left[\begin{array}{l} J_x^{\ a}(\alpha,\beta) + e^{-\sqrt{\alpha^2+\beta^2}(2a)} J_x^{\ -a}(\alpha,\beta) + \\ e^{-\sqrt{\alpha^2+\beta^2}(b-a)} J_x^{\ b}(\alpha,\beta e^{-\sqrt{\alpha^2+\beta^2}(b+a)} J_x^{\ -b}(\alpha,\beta) \end{array} \right] \ for\ y=a \qquad (26)$$

$$A_z = \frac{\mu_0}{8\pi^2} \int \int_{-\infty}^{+\infty} d\alpha d\beta \frac{1}{\sqrt{\alpha^2+\beta^2}} e^{i\alpha x + i\beta z} \left(-\frac{\alpha}{\beta}\right) \left[\begin{array}{l} J_z^{\ a}(\alpha,\beta) + e^{-\sqrt{\alpha^2+\beta^2}(2a)} J_z^{\ -a}(\alpha,\beta) + \\ e^{-\sqrt{\alpha^2+\beta^2}(b-a)} J_z^{\ b}(\alpha,\beta e^{-\sqrt{\alpha^2+\beta^2}(b+a)} J_z^{\ -b}(\alpha,\beta) \end{array} \right] \ for\ y=a \qquad (27)$$

$$A_x = \frac{\mu_0}{8\pi^2} \int \int_{-\infty}^{+\infty} d\alpha d\beta \frac{1}{\sqrt{\alpha^2+\beta^2}} e^{i\alpha x + i\beta z} \left[\begin{array}{l} e^{-\sqrt{\alpha^2+\beta^2}2a} J_x^{\ a}(\alpha,\beta) + J_x^{\ -a}(\alpha,\beta) + \\ e^{-\sqrt{\alpha^2+\beta^2}(b+a)} J_x^{\ b}(\alpha,\beta) + e^{-\sqrt{\alpha^2+\beta^2}(b-a)} J_x^{\ -b}(\alpha,\beta) \end{array} \right] \ for\ y=-a \qquad (28)$$

$$A_z = \frac{\mu_0}{8\pi^2} \int \int_{-\infty}^{+\infty} d\alpha d\beta \frac{1}{\sqrt{\alpha^2+\beta^2}} e^{i\alpha x + i\beta z} \left(-\frac{\alpha}{\beta}\right) \left[\begin{array}{l} e^{-\sqrt{\alpha^2+\beta^2}2a} J_z^{\ a}(\alpha,\beta) + J_z^{\ -a}(\alpha,\beta) + \\ e^{-\sqrt{\alpha^2+\beta^2}(b+a)} J_z^{\ b}(\alpha,\beta) + e^{-\sqrt{\alpha^2+\beta^2}(b-a)} J_z^{\ -b}(\alpha,\beta) \end{array} \right] \ for\ y=-a \qquad (29)$$

$$A_x = \frac{\mu_0}{8\pi^2} \int \int_{-\infty}^{+\infty} d\alpha d\beta \frac{1}{\sqrt{\alpha^2+\beta^2}} e^{i\alpha x + i\beta z} \left[\begin{array}{l} e^{-\sqrt{\alpha^2+\beta^2}(b-a)} J_x^{\ a}(\alpha,\beta) + e^{\sqrt{\alpha^2+\beta^2}(b+a)} J_x^{\ -a}(\alpha,\beta) + \\ J_x^{\ b}(\alpha,\beta) + e^{\sqrt{\alpha^2+\beta^2}(2b)} J_x^{\ -b}(\alpha,\beta) \end{array} \right] \ for\ y=b \qquad (30)$$

$$A_z = \frac{\mu_0}{8\pi^2} \int \int_{-\infty}^{+\infty} d\alpha d\beta \frac{1}{\sqrt{\alpha^2+\beta^2}} e^{i\alpha x + i\beta z} \left(-\frac{\alpha}{\beta}\right) \left[\begin{array}{l} e^{-\sqrt{\alpha^2+\beta^2}(b-a)} J_z^{\ a}(\alpha,\beta) + e^{\sqrt{\alpha^2+\beta^2}(b+a)} J_z^{\ -a}(\alpha,\beta) + \\ J_z^{\ b}(\alpha,\beta) + e^{-\sqrt{\alpha^2+\beta^2}(2b)} J_z^{\ -b}(\alpha,\beta) \end{array} \right] \ for\ y=b \qquad (31)$$

$$A_x = \frac{\mu_0}{8\pi^2} \int \int_{-\infty}^{+\infty} d\alpha d\beta \frac{1}{\sqrt{\alpha^2+\beta^2}} e^{i\alpha x + i\beta z} \left[\begin{array}{l} e^{\sqrt{\alpha^2+\beta^2}(b+a)} J_x^{\ a}(\alpha,\beta) + e^{-\sqrt{\alpha^2+\beta^2}(b+a)} J_x^{\ -a}(\alpha,\beta) + \\ e^{\sqrt{\alpha^2+\beta^2}(2b)} J_x^{\ b}(\alpha,\beta) + J_x^{\ -b}(\alpha,\beta) \end{array} \right] \ for\ y=-b \qquad (32)$$

$$A_z = \frac{\mu_0}{8\pi^2} \int \int_{-\infty}^{+\infty} d\alpha d\beta \frac{1}{\sqrt{\alpha^2+\beta^2}} e^{i\alpha x + i\beta z} \left(-\frac{\alpha}{\beta}\right) \left[\begin{array}{l} e^{\sqrt{\alpha^2+\beta^2}(b+a)} J_z^{\ a}(\alpha,\beta) + e^{-\sqrt{\alpha^2+\beta^2}(b+a)} J_z^{\ -a}(\alpha,\beta) + \\ e^{\sqrt{\alpha^2+\beta^2}(2b)} J_z^{\ b}(\alpha,\beta) + J_z^{\ -b}(\alpha,\beta) \end{array} \right] \ for\ y=-b \qquad (33)$$

The expression for the magnetic field is:

$$B_z = -\partial_z A_y = -\frac{\mu_0}{8\pi^2} \int \int_{-\infty}^{+\infty} d\alpha d\beta e^{i\alpha x + i\beta z} \left[\begin{array}{l} e^{-\sqrt{\alpha^2+\beta^2}(a-y)} J_x^{\ a}(\alpha,\beta) - e^{-\sqrt{\alpha^2+\beta^2}(a+y)} J_x^{\ -a}(\alpha,\beta) + \\ e^{-\sqrt{\alpha^2+\beta^2}(b-y)} J_x^{\ b}(\alpha,\beta) - e^{-\sqrt{\alpha^2+\beta^2}(b+y)} J_x^{\ -b}(\alpha,\beta) \end{array} \right] \ for\ |y| \leq a \qquad (34)$$

$$B_z = -\partial_z A_y = -\frac{\mu_0}{8\pi^2} \int \int_{-\infty}^{+\infty} d\alpha d\beta e^{i\alpha x + i\beta z} \left[\begin{array}{l} -e^{\sqrt{\alpha^2+\beta^2}(y-a)} J_x^{\ a}(\alpha,\beta) + e^{\sqrt{\alpha^2+\beta^2}(a+y)} J_x^{\ -a}(\alpha,\beta) - \\ e^{\sqrt{\alpha^2+\beta^2}(y-b)} J_x^{\ b}(\alpha,\beta) + e^{\sqrt{\alpha^2+\beta^2}(b+y)} J_x^{\ -b}(\alpha,\beta) \end{array} \right] \ for\ |y| \geq b \qquad (35)$$

Transverse Y Gradient Coil

1 For the Y gradient coil, the gradient magnetic field is symmetric along the x and z directions

2 while it is asymmetric along the y direction. In this case the relationship between the current

3 densities on the adjacent planes is:

$$J_x^a(\alpha,\beta) = J_x^{-a}(\alpha,\beta) \quad and \quad J_x^b(\alpha,\beta) = J_x^{-b}(\alpha,\beta) \quad 4$$

5

6 The expression of the magnetic field from equations (34) and (35) becomes:

$$B_z = -\frac{\mu_0}{8\pi^2} \int \int_{-\infty}^{+\infty} d\alpha d\beta e^{i\alpha x + i\beta z} \begin{bmatrix} J_x^a(\alpha,\beta)(e^{-\sqrt{\alpha^2+\beta^2}(a-y)} - e^{-\sqrt{\alpha^2+\beta^2}(a+y)}) + \\ J_x^b(\alpha,\beta)(e^{-\sqrt{\alpha^2+\beta^2}(b-y)} - e^{-\sqrt{\alpha^2+\beta^2}(b+y)}) \end{bmatrix} \quad for\,|y| \le a \qquad (36)$$

$$B_z = -\partial_z A_y = -\frac{\mu_0}{8\pi^2} \int \int_{-\infty}^{+\infty} d\alpha d\beta e^{i\alpha x + i\beta z} \begin{bmatrix} J_x^a(\alpha,\beta)(-e^{-\sqrt{\alpha^2+\beta^2}(y-a)} + e^{\sqrt{\alpha^2+\beta^2}(a+y)}) - \\ J_x^b(\alpha,\beta)(-e^{-\sqrt{\alpha^2+\beta^2}(y-b)} + e^{\sqrt{\alpha^2+\beta^2}(b+y)}) \end{bmatrix} \quad for\,|y| \ge b \qquad (37)$$

7

8 The gradient field shielding condition in the region outside both planes demands that $B_z=0$ for

9 |y|≥b. Equation (37) leads to:

10

$$J_x^{-b'}(\alpha,\beta) = -e^{-\sqrt{\alpha^2+\beta^2}(b'-a')} J_x^{-a'}(\alpha,\beta) \qquad (38)$$

which leads to :

$$B_z = -\frac{\mu_0}{4\pi^2} \int \int_{-\infty}^{+\infty} d\alpha d\beta \cos(\alpha x) \cos(\beta z) e^{-\sqrt{\alpha^2+\beta^2}a} \sinh(y\sqrt{\alpha^2+\beta^2}) J_x^a(\alpha,\beta)\left[1 - e^{-2\sqrt{\alpha^2+\beta^2}(b-a)}\right] \quad (39)$$

and

$$J_x^a(\alpha,\beta) = 4 \int \int_0^\infty dx\,dz \, \cos(\alpha x) \cos(\beta y) J_x^a(x,z) \qquad (40)$$

11 Employing the current density's symmetry conditions, the shielding condition and the continuity

$$W_M = \frac{1}{2} \int_V d^3\vec{x} \ \vec{A} \cdot \vec{J} = \frac{\mu_0}{8\pi^2} \int \int_{-\infty}^{+\infty} d\alpha\,d\beta \frac{\sqrt{\alpha^2+\beta^2}}{\beta^2} |J_x^a(\alpha,\beta)|^2 \left(1 + e^{-2a\sqrt{\alpha^2+\beta^2}}\right)\left[1 - e^{-2\sqrt{\alpha^2+\beta^2}(b-a)}\right] \quad (41)$$

12 equation, the expression for the stored magnetic energy is given by:

13

14 In order to find the optimum continuous current density, which will generate a gradient magnetic

15 field in predetermined locations inside the imaging volume, can be extracted via a Lagrange

16 optimization technique, According to this technique, a functional expression, which is a

17 quadratic function of the current density is constructed. An example of such an expression

1 includes which includes the stored magnetic energy and the magnetic field, and can have the
2 form:

$$F(J_x^a) = W_m - \sum_{j=1}^{N} \lambda_j \left[B_z(\vec{r}_j) - B_{zsc}(\vec{r}_j) \right] \qquad (42)$$

3 where λ_j are the Lagrange multipliers and $B_{zsc}(r_j)$ are the constrained values of the magnetic field
4 inside the desired imaging volume. Minimizing F with respect to J_x^a, a linear matrix equation for
5 J_x^a is obtained:

$$J_x^a = -\frac{\beta^2}{\sqrt{\alpha^2 + \beta^2}} \frac{e^{-a\sqrt{\alpha^2+\beta^2}}}{(1 + e^{-2a\sqrt{\alpha^2+\beta^2}})} \sum_{j=1}^{N} \lambda_j \cos(\alpha x_j) \cos(\beta z_j) \sinh(y_j \sqrt{\alpha^2 + \beta^2}) \qquad (43)$$

6 and the Lagrange multipliers are determined via the constrained equation :

$$\sum_{j=1}^{N} C_{ij} \lambda_j = B_{zsc_i} \Rightarrow$$

$$C_{ij} = \frac{\mu_0}{\pi^2} \int \int_{}^{+\infty} d\alpha\, d\beta \, \cos(\alpha x_i) \cos(\beta z_i) \sinh(y_i \sqrt{\alpha^2 + \beta^2}) \frac{\beta^2}{\sqrt{\alpha^2 + \beta^2}} \frac{e^{-a\sqrt{\alpha^2+\beta^2}} e^{-(b-a)\sqrt{\alpha^2+\beta^2}}}{\cosh(a\sqrt{\alpha^2 + \beta^2})} (44)$$

$$\sinh((b-a)\sqrt{\alpha^2 + \beta^2}) \sum_{j=1}^{N} \lambda_j \cos(\alpha x_j) \cos(\beta z_j) \sinh(y_j \sqrt{\alpha^2 + \beta^2})$$

And the expression for the stored energy is:

$$W_m = \frac{\mu_0}{\pi^2} \int \int_{}^{+\infty} d\alpha\, d\beta \, \frac{\beta^2}{\sqrt{\alpha^2 + \beta^2}} \frac{e^{-b\sqrt{\alpha^2+\beta^2}}}{\cosh(a\sqrt{\alpha^2 + \beta^2})} \sinh((b-a)\sqrt{\alpha^2 + \beta^2}) \qquad (45)$$

$$\left(\sum_{j=1}^{N} \lambda_j \cos(\alpha x_j) \cos(\beta z_j) \sinh(y_j \sqrt{\alpha^2 + \beta^2}) \right)^2$$

11
12
13 **For the case of a non-shielded biplanar transverse gradient set, the above expressions are**
14 **the same at the limit when b→∞.**
15

1   From the continuous current distribution and using employing the stream function technique, a

2   discrete set of loops that closely approximates the continuous current density pattern is

3   generated. In order to verify the accuracy of the discretization, the gradient field of the coil is re-

4   evaluated by applying the Biot-Savart law to the discrete current distribution.

5

6   **Transverse X Gradient Coil**

7

8   For the X gradient coil, the gradient magnetic field is symmetric along the y and z directions

9   while it is asymmetric along the x direction. In this case the relationship between the current

10   densities on the adjacent planes is:

11    $J_x^{a}(\alpha,\beta) = -J_x^{-a}(\alpha,\beta)$ and $J_x^{b}(\alpha,\beta) = -J_x^{-b}(\alpha,\beta)$

12   The expression of the magnetic field from equations (34) and (35) becomes:

$$B_z = -\frac{\mu_0}{8\pi^2}\int\int_{-\infty}^{+\infty}d\alpha d\beta e^{i\alpha x+i\beta z}\left[\begin{array}{l}J_x^{a}(\alpha,\beta)(e^{-\sqrt{\alpha^2+\beta^2}(a-y)}+e^{-\sqrt{\alpha^2+\beta^2}(a+y)})+\\ J_x^{b}(\alpha,\beta)(e^{-\sqrt{\alpha^2+\beta^2}(b-y)}+e^{-\sqrt{\alpha^2+\beta^2}(b+y)})\end{array}\right] \quad for|y|\leq a \qquad (46)$$

$$B_z = -\partial_z A_y = -\frac{\mu_0}{8\pi^2}\int\int_{-\infty}^{+\infty}d\alpha d\beta e^{i\alpha x+i\beta z}\left[\begin{array}{l}J_x^{a}(\alpha,\beta)(-e^{-\sqrt{\alpha^2+\beta^2}a}2\cosh(y\sqrt{\alpha^2+\beta^2}))-\\ J_x^{b}(\alpha,\beta)(-e^{-b\sqrt{\alpha^2+\beta^2}}2\cosh(y\sqrt{\alpha^2+\beta^2}))\end{array}\right] \quad for|y|\geq b \quad (47)$$

14   The gradient field shielding condition in the region outside both planes demands that $B_z=0$ for

15    $|y|\geq b$. Equation (47) leads to:

16

$$J_x^{-b'}(\alpha,\beta) = -e^{-\sqrt{\alpha^2+\beta^2}(b'-a')}J_x^{-a'}(\alpha,\beta) \qquad (48)$$

whichleadsto:

$$B_z = -\frac{\mu_0}{4\pi^2}\int\int_{-\infty}^{+\infty}d\alpha d\beta \sin(\alpha x)\cos(\beta z)e^{-\sqrt{\alpha^2+\beta^2}a}\cosh(y\sqrt{\alpha^2+\beta^2})\hat{J}_x^{a}(\alpha,\beta)\left[1-e^{-2\sqrt{\alpha^2+\beta^2}(b-a)}\right] \qquad (49)$$

and

$$J_x^{a}(\alpha,\beta) = i\hat{J}_x^{a}(\alpha,\beta) \Rightarrow \hat{J}_x^{a}(\alpha,\beta) = 4\int\int_0^{\infty}dx\,dz\,\sin(\alpha x)\cos(\beta z)J_x^{a}(x,z) \qquad (50)$$

17   Employing the current density's symmetry conditions, the shielding condition and the continuity

$$W_M = \frac{1}{2}\int_V d^3\bar{x}\ \bar{A}\bullet\bar{J} = \frac{\mu_0}{8\pi^2}\int\int_{-\infty}^{+\infty}d\alpha\,d\beta\frac{\sqrt{\alpha^2+\beta^2}}{\beta^2}|J_x^{a}(\alpha,\beta)|^2\left(1-e^{-2a\sqrt{\alpha^2+\beta^2}}\right)\left[1-e^{-2\sqrt{\alpha^2+\beta^2}(b-a)}\right] \quad (51)$$

1 equation, the expression for the stored magnetic energy is given by:

2

3 In order to find the optimum continuous current density, which will generate a gradient magnetic

4 field in predetermined locations inside the imaging volume, can be extracted via a Lagrange

5 optimization technique, According to this technique, a functional expression, which is a

6 quadratic function of the current density is constructed. An example of such an expression

7 includes which includes the stored magnetic energy and the magnetic field, and can have the

8 form:

$$F(J_x^a) = W_m - \sum_{j=1}^{N} \lambda_j \left[B_z(\vec{r}_j) - B_{zsc}(\vec{r}_j) \right] \qquad (42)$$

9 where λ_j are the Lagrange multipliers and $B_{zsc}(r_j)$ are the constrained values of the magnetic field

10 inside the desired imaging volume. Minimizing F with respect to J_x^a , a linear matrix equation for

11 J_x^a is obtained:

$$J_x^a = \frac{\beta^2}{\sqrt{\alpha^2 + \beta^2}} \frac{e^{-a\sqrt{\alpha^2 + \beta^2}}}{(1 - e^{-2a\sqrt{\alpha^2 + \beta^2}})} \sum_{j=1}^{N} \lambda_j \sin(\alpha x_j) \cos(\beta z_j) \cosh(y_j \sqrt{\alpha^2 + \beta^2}) \qquad (52)$$

12 and the Lagrange multipliers are determined via the constrained equation :

$$C_{ij} = \frac{\mu_0}{\pi^2} \int \int^{+\infty} d\alpha d\beta \sin(\alpha x_i) \cos(\beta z_i) \cosh(y_i \sqrt{\alpha^2 + \beta^2}) \frac{\beta^2}{\sqrt{\alpha^2 + \beta^2}} \qquad (53)$$

$$\frac{e^{-a\sqrt{\alpha^2 + \beta^2}} [1 - e^{-2(b-a)\sqrt{\alpha^2 + \beta^2}}]}{(1 - e^{-2a\sqrt{\alpha^1 + \beta^1}})} \sum_{j=1}^{N} \lambda_j \sin(\alpha x_j) \cos(\beta z_j) \cosh(y_j \sqrt{\alpha^2 + \beta^2})$$

14

15 **For the case of a non-shielded biplanar transverse gradient set, the above expressions are**

16 **the same at the limit when b→∞.**

17

18 From the continuous current distribution and using employing the stream function technique, a

19 discrete set of loops that closely approximates the continuous current density pattern is

1   generated. In order to verify the accuracy of the discretization, the gradient field of the coil is re-

2   evaluated by applying the Biot-Savart law to the discrete current distribution.

3

4   **<u>Transverse Z Gradient Coil</u>**

5

6   For the Z gradient coil, the gradient magnetic field is symmetric along the y and x directions

7   while it is asymmetric along the z direction. In this case the relationship between the current

8   densities on the adjacent planes is:

9    $J_x^{a}(\alpha,\beta) = -J_x^{-a}(\alpha,\beta)$ and $J_x^{b}(\alpha,\beta) = -J_x^{-b}(\alpha,\beta)$ (54)

10   The expression of the magnetic field from equations (34) and (35) becomes:

$$B_z = -\frac{\mu_0}{8\pi^2}\int\int_{-\infty}^{+\infty} d\alpha d\beta e^{i\alpha x + i\beta z}\begin{bmatrix}J_x^{a}(\alpha,\beta)(e^{-\sqrt{\alpha^2+\beta^2}(a-y)} + e^{-\sqrt{\alpha^2+\beta^2}(a+y)}) + \\ J_x^{b}(\alpha,\beta)(e^{-\sqrt{\alpha^2+\beta^2}(b-y)} + e^{-\sqrt{\alpha^2+\beta^2}(b+y)})\end{bmatrix} \quad for|y|\le a \qquad (55)$$

$$B_z = -\partial_z A_y = -\frac{\mu_0}{8\pi^2}\int\int_{-\infty}^{+\infty} d\alpha d\beta e^{i\alpha x + i\beta z}\begin{bmatrix}J_x^{a}(\alpha,\beta)(-e^{\sqrt{\alpha^2+\beta^2}a}2\cosh(y\sqrt{\alpha^2+\beta^2})) - \\ J_x^{b}(\alpha,\beta)(-e^{b\sqrt{\alpha^2+\beta^2}}2\cosh(y\sqrt{\alpha^2+\beta^2}))\end{bmatrix} \quad for|y|\ge b \quad (56)$$

12   The gradient field shielding condition in the region outside both planes demands that B$_z$=0 for

13 |y|≥b. Equation (56) leads to:

$$J_x^{-b'}(\alpha,\beta) = -e^{-\sqrt{\alpha^2+\beta^2}(b'-a')}J_x^{-a'}(\alpha,\beta) \qquad (57)$$

whichleadsto:

$$B_z = \frac{\mu_0}{4\pi^2}\int\int_{-\infty}^{+\infty}d\alpha d\beta \cos(\alpha x)\sin(\beta z)e^{-\sqrt{\alpha^2+\beta^2}a}\cosh(y\sqrt{\alpha^2+\beta^2})\hat{J}_x^{a}(\alpha,\beta)\left[1-e^{-2\sqrt{\alpha^2+\beta^2}(b-a)}\right] \quad (58)$$

and

$$J_x^{a}(\alpha,\beta) = i\hat{J}_x^{a}(\alpha,\beta) \Rightarrow \hat{J}_x^{a}(\alpha,\beta) = -4i\int\int_0^{\infty}dx\,dz\,\cos(\alpha x)\sin(\beta z)J_x^{a}(x,z) \qquad (59)$$

15   Employing the current density's symmetry conditions, the shielding condition and the continuity

$$W_M = \frac{1}{2}\int_V d^3\bar{x}\ \bar{A}\bullet\bar{J} = \frac{\mu_0}{8\pi^2}\int\int_{-\infty}^{+\infty}d\alpha\,d\beta\frac{\sqrt{\alpha^2+\beta^2}}{\beta^2}|J_x^{a}(\alpha,\beta)|^2\left(1-e^{-2a\sqrt{\alpha^2+\beta^2}}\right)\left[1-e^{-2\sqrt{\alpha^2+\beta^2}(b-a)}\right] \quad (51)$$

16   equation, the expression for the stored magnetic energy is given by:

17

1   In order to find the optimum continuous current density, which will generate a gradient magnetic

2   field in predetermined locations inside the imaging volume, can be extracted via a Lagrange

$$F(J_x^{\ a}) = W_m - \sum_{j=1}^{N} \lambda_j \left[B_z(\vec{r}_j) - B_{zsc}(\vec{r}_j) \right] \qquad (42)$$

3   optimization technique, According to this technique, a functional expression, which is a

4   quadratic function of the current density is constructed. An example of such an expression

5   includes which includes the stored magnetic energy and the magnetic field, and can have the

6   form:

7   where λ_j are the Lagrange multipliers and $B_{zsc}(r_j)$ are the constrained values of the magnetic field

8   inside the desired imaging volume. Minimizing F with respect to $J_x^{\ a}$, a linear matrix equation for

9     $J_x^{\ a}$ is obtained:

$$J_x^{\ a} = \frac{\beta^2}{\sqrt{\alpha^2 + \beta^2}} \frac{e^{-a\sqrt{\alpha^2+\beta^2}}}{(1 - e^{-2a\sqrt{\alpha^2+\beta^2}})} \sum_{j=1}^{N} \lambda_j \cos(\alpha x_j) \sin(\beta z_j) \cosh(y_j \sqrt{\alpha^2 + \beta^2}) \qquad (60)$$

10   and the Lagrange multipliers are determined via the constrained equation :

$$C_{ij} = \frac{\mu_0}{\pi^2} \int \int^{+\infty} d\alpha d\beta \cos(\alpha x_i) \sin(\beta z_i) \cosh(y_i \sqrt{\alpha^2 + \beta^2}) \frac{\beta^2}{\sqrt{\alpha^2 + \beta^2}} \qquad (61)$$

$$\frac{e^{-a\sqrt{\alpha^2+\beta^2}}[1 - e^{-2(b-a)\sqrt{\alpha^2+\beta^2}}]}{(1 - e^{-2a\sqrt{\alpha^1+\beta^1}})} \sum_{j=1}^{N} \lambda_j \cos(\alpha x_j) \sin(\beta z_j) \cosh(y_j \sqrt{\alpha^2 + \beta^2})$$

11

12

13   **For the case of a non-shielded biplanar transverse gradient set, the above expressions are**

14   **the same at the limit when b→∞.**

15

16   From the continuous current distribution and using employing the stream function technique, a

17   discrete set of loops that closely approximates the continuous current density pattern is

18   generated. In order to verify the accuracy of the discretization, the gradient field of the coil is re-

19   evaluated by applying the Biot-Savart law to the discrete current distribution.

20

18

1   Mutual Inductance evaluations for Biplanar Open Architecture coils

2

3   Let us assume a second biplanar gradient set with plane gaps for the primary and secondary coils

4   at locations of $z=2\bar{a}$ and $z=2\bar{b}$, respectively. If the second set is shifted along the x or z

5   directions or both by an amount of x_0 and z_0, respectively, the analytical expression of the current

6   density distribution is given by:

7

8    $$J_x^{-\bar{a},-\bar{b}}(\alpha,\beta) = e^{-i\beta z_0}e^{-i\alpha x_0} \int \int_{-\infty}^{\infty} dx\, dz\, e^{-i\beta(z-z_0)}e^{-i\alpha(x-x_0)}\, J_x^{-\bar{a},-\bar{b}}(x,z) \qquad (62)$$

9

10   Considering for simplicity of the evaluations, that the second set is shifted only along the z

11   direction by an amount of z_0. In this situation the mutual energy between these two modules is

12   given by:

13

$$W_{Mutual} = \frac{1}{2}\int_V d^3\bar{x}\ \ \vec{A}^{(\bar{a},\bar{b})} \bullet \vec{J}^{(\bar{a}',\bar{b})} = \frac{\mu_0}{4\pi^2}\int\int_{-\infty}^{+\infty} d\alpha\, d\beta\, \frac{\sqrt{\alpha^2+\beta^2}}{\beta^2}e^{-i\beta z_0}$$

$$\left\{ \begin{array}{l} \left[\left(e^{-\sqrt{\alpha^2+\beta^2}(a-\bar{a})} - e^{-2\sqrt{\alpha^2+\beta^2}(b-(a+\bar{a}))}\right)\left(J_x^{a}(\alpha,\beta)J_x^{\bar{a}*}(\alpha,\beta) + J_x^{-a}(\alpha,\beta)J_x^{-\bar{a}*}(\alpha,\beta)\right)\right] + \\ \left[\left(e^{-\sqrt{\alpha^2+\beta^2}(a+\bar{a})} - e^{-2\sqrt{\alpha^2+\beta^2}(b-(a-\bar{a}))}\right)\left(J_x^{-a}(\alpha,\beta)J_x^{\bar{a}*}(\alpha,\beta) + J_x^{a}(\alpha,\beta)J_x^{-\bar{a}*}(\alpha,\beta)\right)\right] \end{array} \right\} \qquad (63)$$

16   **Transverse Y gradient Coil**

17

18   Considering the current density symmetries amongst the planes of the self-shielded biplanar

19   gradient coil design, as well as that the mutual energy is evaluated over two gradient coil

20   representing the same axis, equation (63) has the form:

$$W_{Mutual} = \frac{\mu_0}{4\pi^2} \int \int_{-\infty}^{+\infty} d\alpha \, d\beta \, \frac{\sqrt{\alpha^2 + \beta^2}}{\beta^2} \cos(\beta z_0) J_x^a(\alpha,\beta) J_x^{\bar{a}*}(\alpha,\beta)$$

$$\left[e^{-\sqrt{\alpha^2+\beta^2}(a-\bar{a})} \left[1 - e^{-2\sqrt{\alpha^2+\beta^2}(b-(a-\bar{a}))} \right] + e^{-\sqrt{\alpha^2+\beta^2}(a+\bar{a})} \left[1 - e^{-2\sqrt{\alpha^2+\beta^2}(b-(a+\bar{a}))} \right] \right] \quad (64)$$

The optimum phased array configuration occurs at the z_0 points where the value of the mutual inductance between the two sets is minimized.

X and Z gradient Coils

Considering the current density symmetries amongst the planes of the self-shielded biplanar gradient coil design, as well as that the mutual energy is evaluated over two gradient coil representing the same axis, equation (63) has the form:

$$W_{Mutual} = \frac{\mu_0}{4\pi^2} \int \int_{-\infty}^{+\infty} d\alpha \, d\beta \, \frac{\sqrt{\alpha^2 + \beta^2}}{\beta^2} \cos(\beta z_0) J_x^a(\alpha,\beta) J_x^{\bar{a}*}(\alpha,\beta)$$

$$\left[e^{-\sqrt{\alpha^2+\beta^2}(a-\bar{a})} \left[1 + e^{-2\sqrt{\alpha^2+\beta^2}(b-(a-\bar{a}))} \right] - e^{-\sqrt{\alpha^2+\beta^2}(a+\bar{a})} \left[1 - e^{-2\sqrt{\alpha^2+\beta^2}(b-(a+\bar{a}))} \right] \right] \quad (65)$$

The optimum phased array configuration occurs at the z_0 points where the value of the mutual inductance between the two sets is minimized.

Design and Results

1)Non-Shielded Biplanar Gradient Coils

       i)   Transverse Y biplanar gradient coil:

      For the non-shielded transverse biplanar gradient coils, where the gap between the two planes is chosen to be 2a=288mm, the constraint points for the minimization algorithm are shown in Table 1. The gradient field at the center of the structure is demanded to be 40 mT/m with 10% on axis linearity at a distance of $\pm$ 130mm from the center of the gradient and 20% of f axis uniformity inside a 25 cm Diameter Spherical Volume (DSV).

Constraints for a Non-Shielded Biplanar Y gradient coil			
Xj in meters	Yj in meters	Zj in meters	Bzscj in Tesla
0.000	0.001	0.000	0.0000400
0.000	0.130	0.000	0.0046800
0.125	0.001	0.000	0.0000320
0.000	0.001	0.125	0.0000320

Table 1

The application of the Lagrange minimization technique to the set of constraints of the Table 1, generates a continuous current distribution for primary coil of the Transverse Non-Shielded Y biplanar coil. With the assist of the stream function technique, the continuous current densities for the two coils can be approximated by a 10 discrete loops with a common current of 287.6 Amps. Table 2 illustrates all the vital characteristics needed for the engineering and manufacturing phase of such a gradient coil. In addition figure 2 shows the on axis field behavior (on-axis linearity is represented by Figure 2a, off-axis uniformity is represented by Figure 2b)of the coil as it has been evaluated by applying the Biot-Savart law to the coil's discrete current pattern.

Non-Shielded Y Biplanar Gradient Coil	
Property	
Gap	288 mm
Gradient Strength at mT/m/100A	13.9 mT/m/100A
Gradient Strength at 500 A	69.5 mT/m
450 A	62.55 mT/m
330 A	45.87 mT/m
Number of Discrete Loops	10
Cu thick. between adjacent loops	> 12.5mm
Total Inductance (cable included)	95μH
Total Resistance(Cable included)	55 mΩ

Rise Time at 2000V/500A	24 μsec
700V/450A	70 μsec
	82 μsec
Slew Rate at 2000V/500A	2865 T/m/sec
700V/450 A	880　T/m/sec
400V/330A	550　T/m/sec
Duty Cycle (RMS Gradient)	
(500 A ref. point)　at 4500 kW	72% of the peak gradient(49.68mT/m)
at 2000 kW	49% of the peak gradient(33.81mT/m)
at 1000 kW	34% of the peak gradient(23.46mT/m)

1　**Table 2**

2

3

4

ii)　　**Transverse X biplanar gradient coil:**

For the non-shielded transverse biplanar gradient coils, where the gap between the two

planes is chosen to be 2a=284mm, the constraint points for the minimization algorithm are

shown in Table 3. The gradient field at the center of the structure is demanded to be 40 mT/m

with 10% on axis linearity at a distance of $\pm$ 130mm from the center of the gradient and 20% of f

axis uniformity inside a 25 cm Diameter Spherical Volume (DSV).

Constraints for a Non-Shielded Biplanar X gradient coil			
Xj in meters	Yj in meters	Zj in meters	Bzscj in Tesla
0.001	0.000	0.000	0.0000400
0.130	0.000	0.000	.0.0046800
0.001	0.125	0.000	0.0000320
0.001	0.000	0.125	0.0000320

10　**Table 3**

11

12　　　　The application of the Lagrange minimization technique to the set of constraints of the

13　Table 3, generates a continuous current distribution for primary coil of the Transverse Non-

14　Shielded X biplanar coil. With the assist of the stream function technique, the continuous current

15　densities for the two coils can be approximated by a 10 discrete loops with a common current of

22

1 282.9 Amps). Table 4 illustrates all the vital characteristics needed for the engineering and

2 manufacturing phase of such a gradient coil. In addition figure 3 shows the on axis field behavior

3 of the coil as it has been evaluated by applying the Biot-Savart law to the coil's discrete current

4 pattern.

Non-Shielded X Biplanar Gradient Coil	
Property	
Gap	284 mm
Gradient Strength at mT/m/100A	15.62 mT/m/100A
Gradient Strength at 500 A	78.1 mT/m
450 A	70.2 mT/m
330 A	51.5 mT/m
Number of Discrete Loops	10
Cu thick. between adjacent loops	> 12.5mm
Total Inductance (cable included)	177µH
Total Resistance(Cable included)	59 mΩ
Rise Time at 2000V/500A	45 µsec
700V/450A	118 µsec
	154 µsec
Linear Slew Rate at 2000V/500A	1738 T/m/sec
700V/450 A	593 T/m/sec
400V/330A	335 T/m/sec
Duty Cycle (RMS Gradient)	
(500 A ref. point) at 4500 kW	68% of the peak gradient(53.11mT/m)
at 2000 kW	45% of the peak gradient(35.37mT/m)
at 1000 kW	32% of the peak gradient(25.0mT/m)

5 **Table 4**

6   **iii)   Axial Z biplanar gradient coil:**

7   For the non-shielded Axial biplanar gradient coils, where the gap between the two planes

8 is chosen to be 2a=292mm, the constraint points for the minimization algorithm are shown in

9 Table 5. The gradient field at the center of the structure is demanded to be 40 mT/m with 10% on

1 axis linearity at a distance of $\pm$ 130mm from the center of the gradient and 20% of f axis

2 uniformity inside a 25 cm Diameter Spherical Volume (DSV).

3

Constraints for a Non-Shielded Z gradient coil			
Xj in meters	Yj in meters	Zj in meters	Bzscj in Tesla
0.000	0.000	0.001	0.0000400
0.000	0.000	0.130	0.0046800
0.125	0.000	0.001	0.0000325
0.000	0.125	0.001	0.0000325

4 **Table 5**

5

6   The application of the Lagrange minimization technique to the set of constraints of the

7 Table 5, generates a continuous current distribution for primary coil of the Axial Non-Shielded Z

8 biplanar coil. With the assist of the stream function technique, the continuous current densities

9 for the two coils can be approximated by a 10 discrete loops with a common current of 298.15

10 Amps. Table 6 illustrates all the vital characteristics needed for the engineering and

11 manufacturing phase of such a gradient coil. In addition figure 4 shows the on axis field behavior

12 of the coil as it has been evaluated by applying the Biot-Savart law to the coil's discrete current

13 pattern.

14

Non-Shielded Z Biplanar Gradient Coil	
Property	
Gap	292 mm
Gradient Strength at mT/m/100A	13.41 mT/m/100A
Gradient Strength at 500 A	67.08 mT/m
450 A	60.37 mT/m
330 A	44.27 mT/m
Number of Discrete Loops	10
Cu thick. between adjacent loops	> 12.5mm
Total Inductance (cable included)	73μH

Total Resistance(Cable included)	61 mΩ
Rise Time at 2000V/500A	19 μsec
700V/450A	49 μsec
400V/330A	64 μsec
Linear Slew Rate at 2000V/500A	3530 T/m/sec
700V/450 A	1235 T/m/sec
400V/330A	698 T/m/sec
Duty Cycle (RMS Gradient) (500 A ref. point) at 4500 kW	61% of the peak gradient(40.66mT/m)
at 2000 kW	40% of the peak gradient(27.1mT/m)
at 1000 kW	29% of the peak gradient(19.2mT/m)

Table 6

2) Shielded Biplanar Gradient Coils

i)Transverse Y biplanar gradient coil :

For the shielded transverse biplanar gradient coils, the gap between the two primary coil's planes is chosen to be 2a=288mm, while the gap between the shielding planes is 2b=337mm. The constraint points for the minimization algorithm are shown in Table 7. The gradient field at the center of the structure is demanded to be 40 mT/m with 10% on axis linearity at a distance of $\pm$ 130mm from the center of the gradient and 20% of f axis uniformity inside a 25 cm Diameter Spherical Volume (DSV).

Constraints for a Shielded Biplanar Y gradient coil			
Xj in meters	Yj in meters	Zj in meters	Bzscj in Tesla
0.000	0.001	0.000	0.0000400
0.000	0.130	0.000	0.0046800
0.125	0.001	0.000	0.0000320
0.000	0.001	0.125	0.0000320

Table 7

The application of the Lagrange minimization technique to the set of constraints of the Table 7, generates a continuous current distribution for primary coil of the Transverse Shielded Y biplanar coil. With the assist of the stream function technique, the continuous current densities

25

1   for the primary coil can be approximated by a 15 discrete loops with a common current of 425.85

2   Amps(figure 5). **For a shielded Y biplanar gradient coil with parabolic returns, the current**

3   **pattern for the primary coil is shown in Figure 6.** while for the secondary coil its continuous

4   current density can be approximated by 10 loops (figure 7b)with the same constant current as the

5   primary coils. **For a shielded Y biplanar gradient coil with parabolic returns, the current**

6   **pattern for the secondary coil is shown in Figure 7b.** Table 8 illustrates all the vital

7   characteristics needed for the engineering and manufacturing phase of such a gradient coil. In

8   addition figure 8 shows the on axis field behavior of the coil as it has been evaluated by applying

9   the Biot-Savart law to the coil's discrete current pattern.

10

Shielded Y Biplanar Gradient Coil	
Property	
Gap primary/shielding	288 mm/337mm
Gradient Strength at mT/m/100A	9.39 mT/m/100A
Gradient Strength at 500 A	46.95 mT/m
450 A	42.26 mT/m
330 A	31.00 mT/m
Number of Discrete Loops	15/10
Cu thick. between adjacent loops	11.4mm/15.8mm
Total Inductance (cable included)	105μH
Total Resistance(Cable included)	95 mΩ
Rise Time at 2000V/500A	27 μsec
700V/450A	72 μsec
400V/330A	94 μsec
Slew Rate at 2000V/500A	1746 T/m/sec
700V/450 A	587 T/m/sec
400V/330A	330 T/m/sec
Duty Cycle (RMS Gradient) (500 A ref. point) at 4500 kW	47% of the peak gradient(22.27mT/m)
at 2000 kW	32% of the peak gradient(14.85mT/m)

at 1000 kW	22% of the peak gradient(10.50mT/m)
% Eddy Currents at 25cm DSV	0.32%

Table 8

iv)   Transverse X biplanar gradient coil:

For the shielded transverse biplanar gradient coils, where the gap between the two primary planes is chosen to be 2a=284mm, while the gap between the shielding planes is 2b=341mm. The constraint points for the minimization algorithm are shown in Table 9. The gradient field at the center of the structure is demanded to be 40 mT/m with 10% on axis linearity at a distance of $\pm$ 130mm from the center of the gradient and 20% of f axis uniformity inside a 25 cm Diameter Spherical Volume (DSV).

Constraints for a Shielded Biplanar X gradient coil			
Xj in meters	Yj in meters	Zj in meters	Bzscj in Tesla
0.001	0.000	0.000	0.0000400
0.130	0.000	0.000	0.0046800
0.001	0.125	0.000	0.0000320
0.001	0.000	0.125	0.0000320

Table 9

The application of the Lagrange minimization technique to the set of constraints of the Table , generates a continuous current distribution for primary coil of the Transverse Shielded X biplanar coil. With the assist of the stream function technique, the continuous current density for the primary coil can be approximated by a 16 discrete loops with a common current of 339.2 Amps(figure 9). The continuous current density of the shielding coil can be also approximated by a set of 11 discrete loops carrying the same current as the primary coil(figure 10). Table 10 illustrates all the vital characteristics needed for the engineering and manufacturing phase of such a gradient coil.

Shielded X Biplanar Gradient Coil	
Property	
Gap primary/shielding	284 mm/341mm
Gradient Strength at mT/m/100A	11.79 mT/m/100A
Gradient Strength at 500 A	59.00 mT/m
450 A	53.10 mT/m
330 A	38.93 mT/m
Number of Discrete Loops	16/11
Cu thick. between adjacent loops	10.8mm/15.8mm
Total Inductance (cable included)	203µH
Total Resistance(Cable included)	103 mΩ
Rise Time at 2000V/500A	53 µsec
700V/450A	140 µsec
400V/330A	183 µsec
Slew Rate at 2000V/500A	1132 T/m/sec
700V/450 A	422 T/m/sec
400V/330A	213 T/m/sec
Duty Cycle (RMS Gradient)	
(500 A ref. point) at 4500 kW	45% of the peak gradient(26.55mT/m)
at 2000 kW	30% of the peak gradient(17.59mT/m)
at 1000 kW	21% of the peak gradient(12.44mT/m)
% Eddy Currents at 25cm DSV	0.37%

1   **Table 10**

2

3   **v)   Axial Z biplanar gradient coil:**

4   For the shielded Axial biplanar gradient coils, where the gap between the two primary

5   planes is chosen to be 2a=292mm. The distance between the two shielding planes is 2b=345mm.

6   The constraint points for the minimization algorithm are shown in Table 11. The gradient field at

7   the center of the structure is demanded to be 40 mT/m with 10% on axis linearity at a distance of

1    $\pm$ 130mm from the center of the gradient and 7.5% of f axis uniformity inside a 25 cm Diameter

2   Spherical Volume (DSV).

3

Constraints for a Shielded Biplanar Z gradient coil			
Xj in meters	Yj in meters	Zj in meters	Bzscj in Tesla
0.000	0.000	0.001	0.0000400
0.000	0.000	0.130	0.0046800
0.125	0.000	0.001	0.0000394
0.000	0.125	0.001	0.0000394

4   **Table 11**

5

6   The application of the Lagrange minimization technique to the set of constraints of the

7   Table 11, generates a continuous current distribution for primary coil of the Axial Shielded Z

8   biplanar coil. With the assist of the stream function technique, the continuous current densities

9   for the primary coil can be approximated by 16 discrete loops with a common current of 420.9

10   Amps(figure 11). the continuous current density for the shielding coil can be approximated by 12

11   discrete loops (figure 12) Table 12 illustrates all the vital characteristics needed for the

12   engineering and manufacturing phase of such a gradient coil.

13

14

Shielded Z Biplanar Gradient Coil	
Property	
Gap primary/shielding	292 mm/345mm
Gradient Strength at mT/m/100A	9.50 mT/m/100A
Gradient Strength at 500 A	47.51 mT/m
450 A	42.76 mT/m
330 A	31.36 mT/m
Number of Discrete Positive Loops	16/12
Cu thick. between adjacent loops	11.0mm/15.8mm
Total Inductance (cable included)	108 μH

Total Resistance(Cable included)	107 mΩ
Rise Time at 2000V/500A	28 μsec
700V/450A	75 μsec
400V/330A	98 μsec
Slew Rate at 2000V/500A	1712 T/m/sec
700V/450 A	573 T/m/sec
400V/330A	320 T/m/sec
Duty Cycle (RMS Gradient) (500 A ref. point) at 4500 kW	44% of the peak gradient(20.68mT/m)
at 2000 kW	29% of the peak gradient(13.79mT/m)
at 1000 kW	21% of the peak gradient(9.75mT/m)
% Eddy Currents at 25cm DSV	0.33%

Table 12

3) Non-Shielded Uniplanar Gradient Coils

    i)   **Transverse X, Y uniplanar gradient coil**

For the non-shielded transverse uniplanar gradient coils, where the gap between the plane from the geometric center of the structure of the gradient coil is chosen to be $-a'=-6mm$, the constraint points for the minimization algorithm are shown in Table 13. The gradient field at the center of the structure is demanded to be 40 mT/m with 10% on axis linearity at a distance of $\pm$ 100mm from the center of the gradient and 20% of f axis uniformity inside a 25 cm Diameter Spherical Volume (DSV).

Constraints for a Non-Shielded Uniplanar transverse(X,Y) gradient coils			
Xj in meters	Yj in meters	Zj in meters	Bzscj in Tesla
0.000	0.001	0.100	0.0000400
0.000	0.060	0.100	0.0024000
0.000	0.200	0.100	0.0072800
0.180	0.001	0.100	0.0000360

0.000	0.001	0.150	0.0000410
0.000	0.001	0.200	0.0000350

Table 13

The application of the Lagrange minimization technique to the set of constraints of the Table 13, generates a continuous current distribution for primary coil of the Transverse Non-Shielded Y uniplanar coil. With the assist of the stream function technique, the continuous current densities for the coil can be approximated by a 10 discrete loops with a common current of 411.2 Amps (figure 13). Table 14 illustrates all the vital characteristics needed for the engineering and manufacturing phase of such a gradient coil. In addition figure 14 shows the on axis field behavior of the coil as it has been evaluated by applying the Biot-Savart law to the coil's discrete current pattern.

Non-Shielded Transverse (X,Y) Uniplanar Gradient Coil	
Property	
Gradient Strength at mT/m/100A	9.73 mT/m/100A
Gradient Strength at 500 A	48.64 mT/m
450 A	43.77 mT/m
330 A	32.11 mT/m
Number of Discrete Loops	10
Cu thick. between adjacent loops	> 10.2mm
Total Inductance (cable included)	295μH
Total Resistance(Cable included)	73 mΩ
Rise Time at 2000V/500A	75 μsec
700V/450A	199 μsec
400V/330A	259 μsec
Slew Rate at 2000V/500A	648 T/m/sec
700V/450 A	220 T/m/sec
400V/330A	124 T/m/sec

Duty Cycle (RMS Gradient)	
(500 A ref. point) at 4500 kW	53.4% of the peak gradient(26 mT/m)
at 2000 kW	36% of the peak gradient(17.33mT/m)
at 1000 kW	25% of the peak gradient(12.26mT/m)

1   **Table 14**

2

3   ii)   **Axial Z Uniplanar gradient coil:**

4   For the non-shielded Axial Uniplanar gradient coil, the gap between the geometric center

5   of the structure of the gradient coil and the planar surface is chosen to be -a'=-6mm. The

6   constraint points for the minimization algorithm are shown in Table 15. The gradient field at the

7   center of the structure is demanded to be 40 mT/m with 20% on axis linearity at a distance of +

8   200mm from the center of the gradient with no-rollover point and 20% of f axis uniformity

9   inside a 20 cm Diameter Spherical Volume (DSV).

10

Constraints for a Non-Shielded Uniplanar Z gradient coil			
Xj in meters	Yj in meters	Zj in meters	Bzscj in Tesla
0.000	0.000	0.031	0.0000400
0.000	0.000	0.231	0.0062800
0.180	0.000	0.031	0.00003600
0.000	0.200	0.031	0.00003200

11   **Table 15**

12

13   The application of the Lagrange minimization technique to the set of constraints of the

14   Table 15 generates a continuous current distribution for primary coil of the Axial Non-Shielded

15   Z uniplanar coil. With the assist of the stream function technique, the continuous current

16   densities for the two coils can be approximated by a 10 discrete loops with a common current of

17   316.1 Amps(figure 15). Table 16 illustrates all the vital characteristics needed for the

18   engineering and manufacturing phase of such a gradient coil.

19

Non-Shielded Z Uniplanar Gradient Coil	
Property	
Gradient Strength at mT/m/100A	12.65 mT/m/100A
Gradient Strength at 500 A	63.25 mT/m
450 A	56.93 mT/m
330 A	41.75 mT/m
Number of Discrete Loops	13
Cu thick. between adjacent loops	> 12.5mm
Total Inductance (cable included)	241μH
Total Resistance(Cable included)	57 mΩ
Rise Time at 2000V/500A	62 μsec
700V/450A	160 μsec
400V/330A	209 μsec
Linear Slew Rate at 2000V/500A	1034 T/m/sec
700V/450 A	354 T/m/sec
400V/330A	200 T/m/sec
Duty Cycle (RMS Gradient)	
(500 A ref. point) at 4500 kW	63% of the peak gradient(40.0mT/m)
at 2000 kW	42% of the peak gradient(26.7mT/m)
at 1000 kW	30% of the peak gradient(18.9mT/m)

1   **Table 16**

2   **4) Shielded Uniplanar Gradient Coils**

3   **i)Transverse (X,Y) uniplanar gradient coil with parabolic cut-offs:**

4   For the shielded transverse biplanar gradient coils, the gap between the geometric center

5   of the entire gradient structure and the planar surface of the primary coil is chosen to be $-a'=-$

6   6mm. The gap between the geometric center of the structure and the planar surface of the

7   shielding coil is chosen to be $-b'=-70$mm. The constraint points for the minimization algorithm

8   are shown in Table 17. The gradient field at the center of the structure is demanded to be 40

9   mT/m with 20% on axis linearity at a distance of

1   + 200mm from the center of the gradient. In addition, the off-axis uniformity of the gradient field

2   is restricted to within 20% from the field's ideal value inside a 25 cm Diameter Spherical

3   Volume (DSV).

Constraints for a Shielded Uniplanar transverse(X,Y) gradient coils			
Xj in meters	Yj in meters	Zj in meters	Bzscj in Tesla
0.000	0.001	0.075	0.0000400
0.000	0.100	0.075	0.0040000
0.000	0.300	0.075	0.0140400
0.180	0.001	0.075	0.0000380
0.000	0.001	0.13	0.0000390
0.000	0.001	0.180	0.0000320

4   **Table 17**

5

6   The application of the Lagrange minimization technique to the set of constraints of the

7   Table 17 generates a continuous current distribution for primary coil of the Transverse Shielded

8   Y biplanar coil. With the assist of the stream function technique, the continuous current densities

9   for the primary coil can be approximated by a 16 discrete loops with a common current of 324.17

10   Amps(figure 16a), while for the secondary coil its continuous current density can be

11   approximated by 8 loops (figure 16b)with the same constant current as the primary coils. Table

12   18 illustrates all the vital characteristics needed for the engineering and manufacturing phase of

13   such a gradient coil. In addition figure 16c shows the on axis field behavior of the coil as it has

14   been evaluated by applying the Biot-Savart law to the coil's discrete current pattern.

15

Shielded Transverse (X,Y) Uniplanar Gradient Coil	
Property	
Gradient Strength at mT/m/100A	12.33 mT/m/100A
Gradient Strength at 500 A	61.69 mT/m
450 A	55.52 mT/m
330 A	40.70 mT/m

Number of Discrete Loops	16/8
Cu thick. between adjacent loops	12.4mm/15.8mm
Total Inductance (cable included)	302μH
Total Resistance(Cable included)	122 mΩ
Rise Time at 2000V/500A	78 μsec
700V/450A	210 μsec
400V/330A	277 μsec
Slew Rate at 2000V/500A	792 T/m/sec
700V/450 A	264 T/m/sec
400V/330A	147 T/m/sec
Duty Cycle (RMS Gradient) (500 A ref. point) at 4500 kW	40% of the peak gradient(24.95mT/m)
at 2000 kW	27% of the peak gradient(16.64mT/m)
at 1000 kW	19% of the peak gradient(11.76mT/m)
% Eddy Currents at 25cm DSV	0.39%

Table 18

iii)   Axial Z biplanar gradient coil:

For the shielded Axial Uniplanar gradient coils, the gap between the geometric center of the entire gradient structure and the planar surface of the primary coil is chosen to be -a′=-6mm. The gap between the geometric center of the structure and the planar surface of the shielding coil is chosen to be -b′=-70mm. The constraint points for the minimization algorithm are shown in Table 19. The gradient field at the center of the structure is demanded to be 40 mT/m with 20% on axis linearity at a distance of ± 130mm from the center of the gradient and 7.5% of f axis uniformity inside a 25 cm Diameter Spherical Volume (DSV).

Constraints for a Shielded Uniplanar Z gradient coil			
Xj in meters	Yj in meters	Zj in meters	Bzscj in Tesla
0.000	0.000	0.031	0.0000400

0.000	0.000	0.231	0.0062800
0.180	0.000	0.031	0.00003600
0.000	0.200	0.031	0.00003200

Table 19

The application of the Lagrange minimization technique to the set of constraints of the Table 19, generates a continuous current distribution for primary coil of the Axial Shielded Z biplanar coil. With the assist of the stream function technique, the continuous current densities for the primary coil can be approximated by 21 discrete loops with a common current of 385.71 Amps(figure 17a). The continuous current density for the shielding coil can be approximated by 13 discrete loops (figure 17b) Table 20 illustrates all the vital characteristics needed for the engineering and manufacturing phase of such a gradient coil.

Shielded Z Uniplanar Gradient Coil	
Property	
Gradient Strength at mT/m/100A	10.37 mT/m/100A
Gradient Strength at 500 A	51.85 mT/m
450 A	46.67 mT/m
330 A	34.22 mT/m
Number of Discrete Positive Loops	21/13
Cu thick. between adjacent loops	9.4mm/15.8mm
Total Inductance (cable included)	306 µH
Total Resistance(Cable included)	125 mΩ
Rise Time at 2000V/500A	80 µsec
700V/450A	213 µsec
400V/330A	281 µsec
Slew Rate at 2000V/500A	656 T/m/sec
700V/450 A	218 T/m/sec

400V/330A	122 T/m/sec
Duty Cycle (RMS Gradient)	
(500 A ref. point) at 4500 kW	40% of the peak gradient(20.97mT/m)
at 2000 kW	27% of the peak gradient(13.98mT/m)
at 1000 kW	19% of the peak gradient(9.89mT/m)
% Eddy Currents at 25cm DSV	0.19%

Table 12

Mutual Inductance Calculations

Biplanar Gradients

The mutual energy between two biplanar gradients of the same axis was evaluated based on the equations (64) and (65). In particular for this example the mutual energy between two biplanar transverse Y gradients was evaluated. For the first biplanar set, the gap between its primary planes was set at $2a$=288mm, while the gap between its shielding planes was set at $2b$=341mm. For the second biplanar set, the gap between the primary planes was set at $2\bar{a}$=282mm, while the gap between the two secondary planes is set at $2\bar{b}$=292mm. The mutual energy of these two sets as a function of shifting one set with respect to the other along the z direction. The optimum (zero) mutual energy of these two sets occurs at the point z_0=216mm from the geometric center of the fir gradient coil set.

Uniplanar Gradients

The mutual energy between two uniplanar gradients of the same axis was evaluated based on the equations (21). In particular for this example the mutual energy between two uniplanar axial Z gradients was evaluated. For the first uniplanar set, the gap on the primary plane was set at $-a'$=-6mm, while the gap of the shielding plane was set at $-b'$=-70mm. For the second biplanar set, the gap for the primary plane was set at $-\bar{a}'$=-3mm, while the gap between the two secondary planes is set at $-\bar{b}'$=-75mm. The optimum (zero) mutual inductance for these two set occurs at the point z_0=245mm from the geometric center of the first gradient coil set.

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2   The present invention describes a method of designing and manufacturing an Open

3   Architecture gradient coil set which is suitable for magnetic resonance systems with vertically

4   directed or horizontally directed fields. The concept of designing of an open architecture gradient

5   coil system offer a significant advantage over existing geometries since, the present gradient

6   design provides superior performance in terms of gradient strength and field linearity and

7   uniformity while the structure of the coil is an open architecture focusing of reducing patient

8   claustrophobia. The invention can be implement in gradient coil designs for open magnet

9   geometries with either vertically and or horizontally directed fields. In particular, for magnets

10   with vertically directed fields, besides the open architecture design of the proposed gradient

11   system a significant gradient increase in the strength of the local gradient field up to 55mT/m

12   with slew rates exceeding 1000 T/m/sec in an imaging volume covering the size of the human

13   head (25 cm DSV). Another significant advantage of the design is the ability to control and

14   minimize the dB/dt and eddy current levels for shielded design. In addition, the open architecture

15   design can be paired in a phased array configuration with alternative shielded or not open

16   architecture gradient coil set in such a configuration that covers the human body head to toe. The

17   significant advantage of such configuration is the significant reduction of dB/dt effects and the

18   utilization of high strength non-claustrophobic gradient coils for high speed imaging of the

19   human body with excellent gradient field uniformity and linearity characteristics in the local

20   predetermined gradient volume. For open magnets with horizontally directed fields (double-

21   donut designs), the invention provides a competitive advantage over existing gradient designs

22   since it can deliver gradient fields with strengths two to three fold grater than already existing

23   designs and the ability to cover the entire human body in a phased array configuration with

24   minimized dB/dt effects and induced eddy currents on the magnet's shields or metallic surfaces

25   during the localization of the gradient field. The open architecture can be modified with

26   parabolic cut-offs in order to incorporate the human shoulders for the portion of the open

27   architecture gradient system which is suitable for imaging the human head. Furthermore, the

28   design can be incorporated as an insertable configuration and can be fully integrated with any

29   volumetric or surface phased array or singular coil configuration for imaging portions or the

30   entire human body. In another aspect of the invention the design can be secured into the structure

31   of its corresponding design. Although the design incorporates a limited curvature for the

1   perspective gradient coil configurations, the curvature can be easily modified for those skilled to

2   the art to any unspecified shape suitable for the any targeted application. In addition, for a self-

3   shielded design the side planes of the secondary (shielded) coil can be mechanically shifted and

4   can be adjusted with respect the primary planes of the coil in an array configuration with zero

5   mutual inductance in such a way that the imaging volume of the coil is extended.

6   The present invention provides a gradient coil set that minimizes patient anxiety while

7   delivering high peak, uniformity and slew rate gradient coils for Large imaging volumes. The

8   combination of both biplanar and uniplanar gradients achieve a gradient magnetic field with

9   superior field qualities. The coil set may have the side planes with parabolic cut offs in order to

10   assist in patient comfort for applications that are focused on imaging the head, the neck, the

11   spine, the heart and the upper torso area. Due to the phased array design low dB/dt levels can be

12   achieved. In the gradient coil set and especially for the shielded designs, the outer shielding

13   planes can be moved to form a phased array configuration with their perspective primary planes

14   corresponding to the same gradient coil axis. The movement can be done via mechanical means

15   or any other means. An unlimited spectrum of FMRI and Interventional applications can be

16   performed with the he "Open Face Architecture" gradient coil, because of its openness and the

17   high peak and slew rate values for the gradient field.

18   Referring to FIG. 1, a gradient coil set 10 includes a uniplanar Z-gradient coil 12 and a

19   biplanar X-gradient coil 14 and a biplanar Y-gradient coil 16. Each of coils may include a

20   respective shield coil 18, 20, 22. The coil set 10 has an open Z-axis face opposite the coil 12.

21   The biplanar coils may be provided with shoulder reliefs 24. The structure of the biplanar coils

22   14, 16 may be conjoined with the structure of the uniplanar coil 12, for example, as shown.

23   Referring to FIG. 18, a gradient coil set 30 includes self-shielded gradient coils 32, 34, 36

24   and a RF head coil 38

25   Referring to FIG. 19, a gradient coil set 40 includes un-shielded gradient coils 42, 44, 46

26   and a RF head coil 48.

27   Referring to FIG. 20, a gradient coil set 50 includes self-shielded gradient coils 52, 54,

28   56, a RF head coil 58 and a CTL spine coil 59 integrated into the structure of the gradient coil set

29   50.

30   Referring to FIG. 21, a gradient coil set 60 includes un-shielded gradient coils 62, 64, 66,

31   a RF head coil 68 and a CTL spine coil 69 integrated into the structure of the gradient coil set 60.

1   Referring to FIG. 22, a gradient coil set 70 includes Z-gradient uniplanar coils 72, 74 and

2   X-Y biplanar gradient coils 76, 78 in a phased array configuration. A head RF coil 79 may be

3   included.

4   Referring to FIG. 23, an open Z-axis face coil set 90 includes a knee and upper thigh RF

5   coil 92 integrated therewith.

6   Referring to FIG. 24, an open Z-axis face coil set 100 includes a knee and foot RF coil

7   102 integrated therewith.

8   It should be evident that this disclosure is by way of example and that various changes

9   may be made by adding, modifying or eliminating details without departing from the fair scope

10   of the teaching contained in this disclosure. The invention is therefore not limited to particular

11   details of this disclosure except to the extent that the following claims are necessarily so limited.

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